

# 半径方向流入水カタービンの性能に及ぼすコリオリカの影響

## The Effect of Coriolis Forces on the Performance of Radial Inflow Hydraulic Turbines

Hans E. KIMMEL (Ebara Int.)

Hans E. KIMMEL, Ebara International Corporation, Sparks, Nevada, USA

The performance of variable speed radial inflow hydraulic turbines is affected by Coriolis forces. With increasing flow and rotational speed, the influence of the Coriolis force on hydraulic efficiency and performance stability becomes more significant. The performance equation for variable speed turbines is based on Newton’s Law of Conservation of Energy and describes the balance between the static and the kinetic fluid energy. To define the influence of each form of energy the kinetic fluid energy is divided into translatory, rotatory and Coriolis energy. A desirable result it is that the Coriolis energy increases the hydraulic efficiency, but it is shown that Coriolis forces also cause the performance curves for constant rotational speed to cross each other, generating undesirable regional instabilities.

**Key Words:** Coriolis force, radial inflow turbine, variable speed, hydraulic efficiency, performance instabilities

### 1. Introduction

In many cryogenic and chemical processes, liquids are often expanded from a high pressure to a lower pressure level. Traditionally the expansion has been achieved by reducing the pressure across a Joule-Thomson Valve where the pressure energy in the fluid is mostly transferred into heat. If the available energy is sufficiently large it is worthwhile to expand the liquid in a hydraulic turbine generator, recovering the fluid energy as electrical power increasing the productivity of the process plant. Cryogenic gas liquefaction processes particularly benefit from the installation of expansion turbines because the liquefaction is a thermodynamic refrigeration process to reduce the enthalpy of the product. The lower the temperature of the product, the higher the economic benefits, due to the fact that refrigeration has a low Carnot efficiency and requires high energy input. A worldwide increase in demand for Liquefied gases is being experienced and the on-going development of cryogenic expansion turbines with higher efficiency is a primary target.

### 2. Variable Speed Turbines

Radial inflow turbines, also known as reaction turbines, Francis or Kaplan turbines, are suitable designs for the expansion of cryogenic liquefied gases, and of other liquids (1). Figure 1 shows a cross section of the turbine runner with the arrows indicating the radial liquid inflow. The liquid passing through the turbine runner has a radial and an angular velocity component.

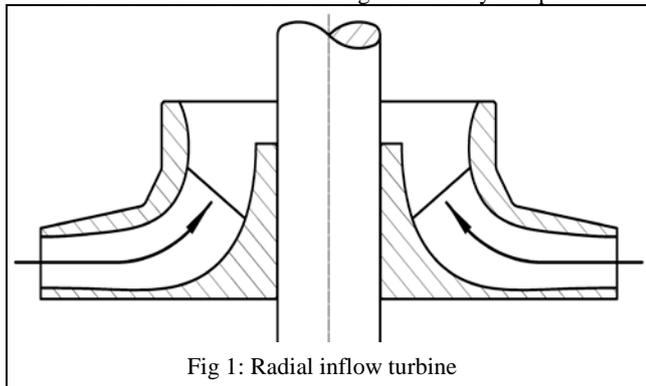


Fig 1: Radial inflow turbine

Figure 2 demonstrates the angular velocity components  $c$ , which determine the generated torque  $T$  according to the Euler Turbine Equation. The torque is proportional to the mass flow  $m$  multiplied by the difference of the angular fluid momentum at the inlet minus the outlet.

$$T \propto m (c_1 r_1 - c_2 r_2)$$

$m$  and  $c_1$  are proportional to  $Q$  and  $c_2$  is proportional to  $N$ . The Euler Turbine Equation satisfies the Conservation of Momentum Law.

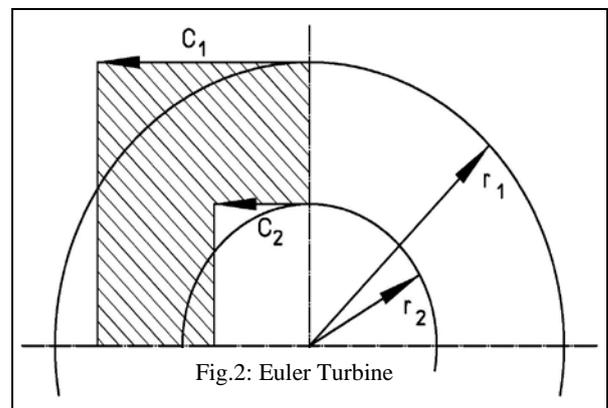


Fig.2: Euler Turbine

Any particle that moves within a rotating system is subject to Coriolis acceleration (3). Figure 3 shows the path  $S$  of a fluid particle  $P$  moving radial with the velocity  $Q$  to the rotation centre with the rotational speed  $N$ . The particle  $P$  is deflected in the rotational direction by the Coriolis force. The Coriolis energy  $E_C$  is a kinetic energy and proportional to the product of  $Q$  and  $N$ , which are both velocities:  $E_C \propto Q N$

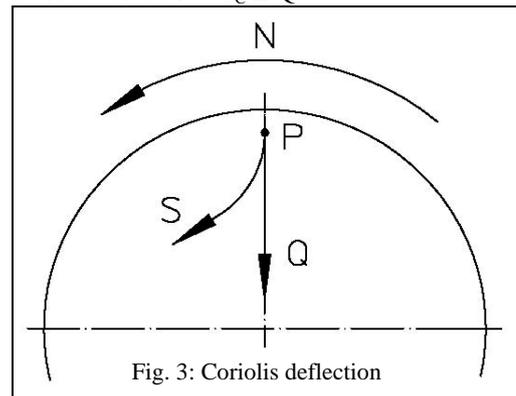


Fig. 3: Coriolis deflection

The Conservation of Energy Law determines the performance of variable speed turbines. With the given differential head  $H$  between turbine inlet and outlet as the static fluid energy, and the three kinetic energies of translatory  $E_T$ , rotatory  $E_R$  and Coriolis energy  $E_C$ , the performance formula becomes

$$H = E_T + E_R + E_C$$

With the kinetic energies:  $E_T = \alpha Q^2$  ;  $E_R = \beta N^2$  ;  
 $E_C = \gamma Q N$  and  $\alpha$ ;  $\beta$ ;  $\gamma$ ; as the scaling constants.

### 3. Tested performance

Allison-Youel (4) conducted extensive tests on cryogenic variable speed radial inflow turbines and generated a detailed performance map. The test points were connected by least square second order

polynomials observing the hydraulic affinity laws. The test result shows that the Coriolis energy represented by the scaling constant  $\gamma$  has a negative value only for radial inflow turbines.

$$H + \gamma Q N = \alpha Q^2 + \beta N^2$$

The Coriolis energy increases the available static energy, and increases the hydraulic efficiency  $\eta$  of the turbine.

$$\eta = (\text{Power output})/(\text{Power input})$$

$$\text{Power input} \propto H Q = H (\alpha Q^2 + \beta N^2 - \gamma Q N)$$

The negative Coriolis energy reduces the available power input and increases the hydraulic efficiency.

#### 4. Performance Instabilities

The performance curves for constant rotational speed  $N$  in the  $Q$ - $N$ -map are shown in Figures 4 and 5.

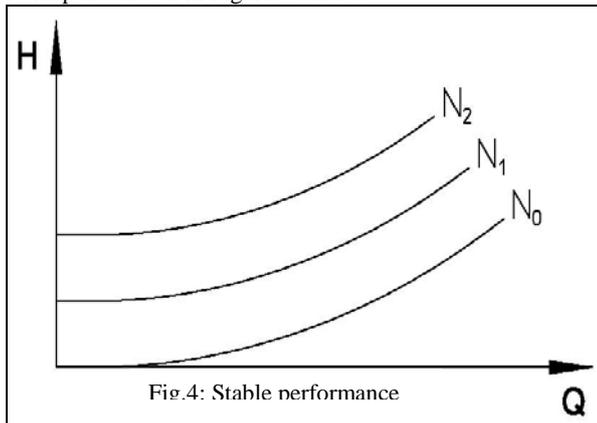


Fig.4: Stable performance

Figure 4 demonstrates the case without the Coriolis energy, and the speed curves are parallel to each other, but shifted to higher differential heads for higher speeds.

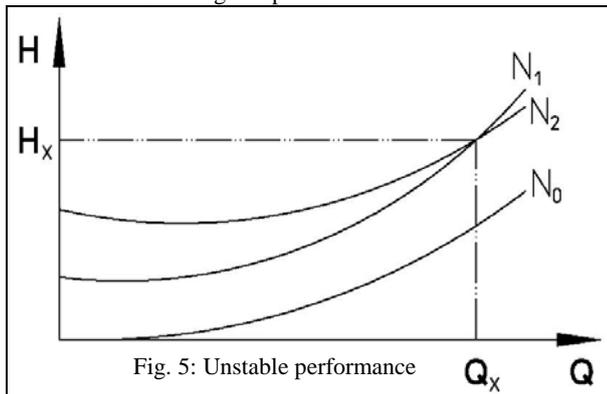


Fig. 5: Unstable performance

Figure 5 with the Coriolis energy included, shows each speed curve increasingly tilted clockwise for increasing rotational speeds. As Figure 5 demonstrates the curves for different rotational speeds are then intersecting each other at certain points [  $Q_x$ ,  $H_x$  ] generating undesirable regional performance instabilities.

The Coriolis energy has no influence on the Euler Turbine Equation and the generated torque, because only the angular velocities determine the torque.

#### 5. Performance Field

Variable speed radial inflow turbines have a limited performance field, consisting of various performance curves depicting the relationship between head and flow at a constant speed, Fig. 4 and Fig. 5.

There are two specific performance curves that border the turbine operating field. The first one is the curve connecting all the points for which the rotational speed is zero. The turbine is not rotating along this curve, shown in Fig.6 as curve  $L_0$ . The second boundary curve shown in Fig.6 is curve  $L_{0C}$  for which the Euler turbine torque is equal to zero. This is the curve corresponding with a no-load condition of the turbine and the hydraulic energy presented to the turbine is entirely dissipated and converted into

friction and heat. The thermodynamic turbine performance along this no-load curve corresponds to the performance of a Joule-Thomson Valve.

All constant speed curves intersect with the no-load curve. At the intersection the rotational speed is the same for the no-load curve and the constant speed curve.

The boundary for turbine operation is between two specific curves: the zero speed curve  $N_0$  and the no-load curve  $L_0$  as shown in Fig.6.

The first condition for the no-load curve is the Euler torque zero, with  $c_1 r_1 - c_2 r_2 = 0$  or  $N r_1 - Q r_2 = 0$  and the second condition is the intersection with the constant speed curves. The constant speed curves are affected by the Coriolis energy, whilst the Euler torque remains independent of the Coriolis energy; hence the intersection is generating a no-load curve which is affected by the Coriolis energy. Fig.6 shows two different no-load curves,  $L_0$  and  $L_{0C}$ . For a turbine designed taking no Coriolis energy into account, the no-load curve  $L_0$  is located at lower flow rates. For a turbine incorporating the Coriolis energy, the no-load curve  $L_{0C}$  is located at high flow rates. Because the zero speed curve  $N_0$  is independent of Coriolis energy, the total available turbine performance field area, between the zero speed and the no-load curve, is smaller if the Coriolis energy is included.

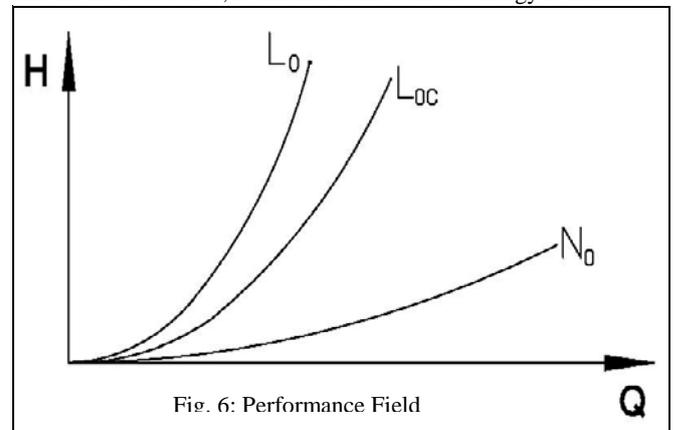


Fig. 6: Performance Field

#### 6. Summary

The affect of the Coriolis energy on variable speed radial inflow turbines is three fold. The Coriolis energy increases the hydraulic efficiency of the turbine, introduces performance instabilities at high rotational speeds and high flow rates, and decreases the available performance field in the  $Q$ - $H$ -map.

#### References

- (1) Kimmel, H.E., "Variable speed turbine generators in LNG liquefaction plants" Gastech 96, The 17<sup>th</sup> International LNG/LPG/Natural Gas Conference and Exhibition, 3-6 December 1996, Vienna, Austria, Conference Papers Volume 2, ISBN 1 874134 16 2
- (2) Kimmel, H.E. "Hydraulic performance of speed controlled turbines for power recovery in cryogenic and chemical processing" World Pumps, Elsevier, Oxford, England, June 1997, pp.46-49
- (3) Vanyo, J.P., "Rotating Fluids in Engineering and Science", Dover Publication 1993, ISBN 0-486-41704-2
- (4) Alison-Youel, S.D., "Observation and analysis of affinity law deviations through tested performance of liquefied gas reaction turbines" The 12<sup>th</sup> International Symposium on Transport Phenomena and Dynamics of Rotating Machinery, ISROMAC 12-2008, Honolulu, Hawaii, February 17-22, 2008
- (5) Habets, G.L.G.M., "Monitoring Cryogenic Turbines Using No-Load Characteristics" ISROMAC-8, Honolulu, HI, USA March 26-30, 2000